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Applications of Quantum Physics

1. Binding Energies

Consider a calcium atom Ca($1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$).

For reference, the binding energy of H($1s$) is 13.6 eV.

a) Calculate the ionization potential (in eV) of this atom. The quantum defect is Δn=2.51.

b) Calculate the effective charge experienced by the 4s electron.

c) The total binding energy of the two 4s electrons together is 18.0 eV. Calculate the quantum defect describing the binding energy of the 4s electron in Ca^+($1s^2 2s^2 2p^6 3s^2 3p^6 4s$).

d) Why is the quantum defect calculated at b) larger, equal or smaller than the one given at a).

2. ^8Li isotopes in magnetic fields

Consider the Li isotope ^8Li which has a nuclear spin of $I = 2$.

Sketch the binding energy of the states belonging to the ground state electronic configuration ($1s^2 2s$) of ^8Li as a function of increasing magnetic field. Indicate the relevant quantum numbers for magnetic field regimes of B=0, B="weak" and B="strong".

3. Hyperfine structure

A $^4P_{5/2}$ level is split by hyperfine interaction into six states that have consecutive separations of 420, 630, 840, 1050 and 1260 MHz.

a) Calculate the hyperfine structure constant, A.

b) Deduce the nuclear spin of the atom.

4. Configurations, Terms, States, and Hund's rules

Consider an atom in the following ground electronic configuration:
$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^2 5s^2 5p^6 6s^2$.

a) Use LS coupling to determine all possible terms and states associated with this electronic configuration

b) Indicate which one is the ground state and sketch the binding energy sequence of the terms and states assuming that the Hund's rules apply to all terms.

5. Quantum teleportation

To pass the feared "applications of quantum physics" exam two fictitious students (Mark and Rita) come up with the devious plan to exchange information via quantum teleportation. One way or the other Mark knows the answer to problem 5, namely that the "answer" qubit (with eigenstates $|0\rangle$ and $|1\rangle$) has the wavefunction $\alpha|0\rangle + \beta|1\rangle$ ($\alpha^2+\beta^2=1$).

To transfer this information to Rita an entangled state of two particles (each with eigenstates $|0\rangle$ and $|1\rangle$) will be used. The entangled state they use is $2^{-½}\{|01\rangle - |10\rangle\}$ of which the first qubit resides with Mark while the second one is send to Rita.

a) Show that the used state $2^{-½}\{|01\rangle - |10\rangle\}$ is indeed entangled

b) Write down the combined wave function of answer and entangled state

c) Determine the wave function after Mark has performed a controlled-NOT operation on the first qubit of the entangled state with the "answer" qubit as control bit. In a controlled-NOT operation the qubit operated on changes state (from $|0\rangle$ to $|1\rangle$ or vice versa) if the control bit is in state $|1\rangle$, if the control bit is in state $|0\rangle$ nothing happens.

d) Next, Mark performs a Hadamard gate operation on the answer qubit. Determine the wave function. The Hadamard gate operation is given by $2^{-½}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

e) Finally Mark performs a measurement and finds $|00\rangle$. Which operation has Rita to perform on her wavefunction to get the correct answer $\alpha|0\rangle + \beta|1\rangle$.